Nama : Mohammad Febryan Khamim
NRP : 5002221072

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Tugas Metode Matematika (D)

$$P_4(x) = \frac{1}{2^4 \cdot 4!} \frac{d^4}{dx^4} (x^2-1)^4$$

Akan dibuktikan $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$

$$P_4(x) = \frac{1}{2^4 \cdot 4!} \frac{d^4}{dx^4} (x^2-1)^4$$

$$= \frac{1}{16 \cdot 4!} \frac{d^3}{dx^3} \left[4(x^2-1)^3 \cdot 2x \right]$$

$$= \frac{1}{16 \cdot 4!} \frac{d^3}{dx^3} \left[8(x^2-1)^3 \cdot x \right]$$

$$= \frac{1}{2 \cdot 4!} \frac{d^2}{dx^2} \left[3(x^2-1)^2 2x \cdot x + (x^2-1)^3 \right]$$

$$= \frac{1}{2 \cdot 4!} \frac{d^2}{dx^2} \left[6x^2(x^2-1)^2 + (x^2-1)^3 \right]$$

$$= \frac{1}{2 \cdot 4!} \frac{d}{dx} \left[12x(x^2-1)^2 + 6x^2 \cdot 2(x^2-1) \cdot 2x + 3(x^2-1)^2 \cdot 2x \right]$$

$$= \frac{1}{2 \cdot 4!} \frac{d}{dx} \left[12x(x^2-1)^2 + 24x^3(x^2-1) + 6x(x^2-1)^2 \right]$$

$$= \frac{3}{4!} \frac{d}{dx} \left[2x(x^2-1)^2 + 4x^3(x^2-1) + x(x^2-1)^2 \right]$$

$$= \frac{3}{4!} \left[2 \cdot (x^2-1)^2 + 2x \cdot 2(x^2-1) \cdot 2x + 12x^2(x^2-1) + 4x^3 \cdot 2x + (x^2-1)^2 + x \cdot 2(x^2-1)2x \right]$$

$$= \frac{3}{4!} \left[2(x^4 - 2x^2 + 1) + 8x^4 - 8x^2 + 12x^4 - 12x^2 + 8x^4 + x^4 - 2x^2 + 1 + 4x^4 - 4x^2 \right]$$

$$= \frac{1}{8} \left[(2+8+12+8+1+4)x^4 + (-4-8-12-2-4)x^2 + (2+1) \right]$$

$$= \frac{1}{8} \left[35x^4 - 30x^2 + 3 \right] \quad [\text{TERBUKTI}]$$

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$P_5(x) = \frac{1}{8}\left(63x^5 - 70x^3 + 15x\right)$; Buktikan !

$$P_5(x) = \frac{1}{2^5.5!} \frac{d^5}{dx^5} (x^2-1)^5$$

$$= \frac{1}{32.5!} \frac{d^4}{dx^4} \left[5(x^2-1)^4 . 2x\right]$$

$$= \frac{1}{16.4!} \frac{d^3}{dx^3} \left[4(x^2-1)^3 . 2x . x + (x^2-1)^4\right]$$

$$= \frac{1}{16.4!} \frac{d^3}{dx^3} \left[8x^2(x^2-1)^3 + (x^2-1)^4\right]$$

$$= \frac{1}{16.4!} \frac{d^2}{dx^2} \left[16x(x^2-1)^3 + 8x^2 . 3(x^2-1)^2 . 2x + 4(x^2-1)^3 . 2x\right]$$

$$= \frac{1}{16.4!} \frac{d^2}{dx^2} \left[16x(x^2-1)^3 + 48x^3(x^2-1)^2 + 8x(x^2-1)^3\right]$$

$$= \frac{1}{2.4!} \frac{d^2}{dx^2} \left[2x(x^2-1)^3 + 6x^3(x^2-1)^2 + x(x^2-1)^3\right]$$

$$= \frac{1}{2.4!} \frac{d}{dx} \left[2(x^2-1)^3 + 2x . 3(x^2-1)^2 . 2x + 18x^2(x^2-1)^2 + 6x^3 . 2(x^2-1) 2x + (x^2-1)^3 + x . 3(x^2-1)^2 . 2x\right]$$

$$= \frac{1}{2.4!} \frac{d}{dx} \left[2(x^2-1)^3 + 12x^2(x^2-1)^2 + 18x^2(x^2-1)^2 + 24x^4(x^2-1) + (x^2-1)^3 + 6x^2(x^2-1)^2\right]$$

$$= \frac{1}{2.4!} \left[6(x^2-1)^2 . 2x + 24x(x^2-1)^2 + 12x^2 . 2(x^2-1) . 2x + 36x(x^2-1)^2 + 18x^2 . 2(x^2-1) . 2x + 96x^3(x^2-1) + 24x^4 . 2x + 3(x^2-1)^2 . 2x + 12x(x^2-1)^2 + 6x^2 . 2(x^2-1) . 2x\right]$$

$$= \frac{1}{2.4!} \left[12x(x^4-2x^2+1) + 24x(x^4-2x^2+1) + 48x^3(x^2-1) + 36x(x^4-2x^2+1) + 72x^3(x^2-1) + 96x^3(x^2-1) + 48x^5 + 6x(x^4-2x^2+1) + 12x(x^4-2x^2+1) + 24x^3(x^2-1)\right]$$

$$= \frac{1}{2.4!} \left[12x^5 - 24x^3 + 12x + 24x^5 - 48x^3 + 24x + 48x^5 - 48x^3 + 36x^5 - 72x^3 + 36x + 72x^5 - 72x^3 + 96x^5 - 96x^3 + 48x^5 + 6x^5 - 12x^3 + 6x + 12x^5 - 24x^3 + 12x + 24x^5 - 24x^3\right]$$

$$= \frac{1}{2.4!} \left[(12+24+48+36+72+96+48+6+12+24)x^5 + (-24-48-48-72-72-96-12-24-24)x^3 + (12+24+36+6+12)x\right]$$

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$$= \frac{1}{2.4!}\left[378x^5 - 420x^3 + 90x\right]$$

$$= \frac{1}{2.4!}\left[6\left(63x^5 - 70x^3 + 15x\right)\right]$$

$$= \frac{1}{2.4.3.2}\left[6\left(63x^5 - 70x^3 + 15x\right)\right]$$

$$= \frac{1}{8}\left[63x^5 - 70x^3 + 15x\right] \quad [\text{TERBUKTI}]$$

Pembuktian Rumus 15

$$(1-x^2)P_n'(x) = n\left[P_{n-1}(x) - xP_n(x)\right]$$

Dari relasi (2) :

$$nP_n(x) = xP_n'(x) - P_{n-1}'(x) \quad \times x$$

$$n\,x\,P_n(x) = x^2P_n'(x) - xP_{n-1}'(x) \quad \ldots\ldots \text{(i)}$$

Dari relasi (4) :

$$(n+1)P_n(x) = P_{n+1}'(x) - xP_n'(x) \quad \text{ubah } n \rightarrow n+1$$

$$nP_{n-1}(x) = P_n'(x) - xP_{n-1}'(x) \ldots \text{(ii)}$$

Dari (i) dan (ii)

$$nP_{n-1}(x) = P_n'(x) - xP_{n-1}'(x)$$

$$nxP_n(x) = x^2P_n'(x) - xP_{n-1}'(x) \quad -$$

$$n\left[P_{n-1}(x) - xP_n(x)\right] = P_n'(x)\left[1-x^2\right]$$

$\hookrightarrow$ Terbukti bahwa $(1-x^2)P_n'(x) = n\left[P_{n-1}(x) - xP_n(x)\right]$

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16). $f(x) = x^4 + 2x^3 + 2x^2 - x + 3$

-> Cari x^4 dari Polinomial Legendre.

※ $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$

Untuk dapat Koef x^4 adalah 1, maka kalikan dengan $\frac{8}{35}$

$$\Rightarrow \frac{8}{35} \cdot \frac{1}{8}(35x^4 - 30x^2 + 3) = x^4 - \frac{30}{35}x^2 + \frac{3}{35}$$

$$= x^4 - \frac{6}{7}x^2 + \frac{3}{35}$$

-> Cari $2x^3$ dari Polinomial Legendre

※ $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$

Maka kalikan $P_3(x)$ dengan $\frac{4}{5}$, diperoleh :

$$\Rightarrow \frac{4}{5}P_3(x) = \frac{4}{5}\left[\frac{5}{2}x^3 - \frac{3}{2}x\right] = 2x^3 - \frac{6}{5}x$$

-> Cari $2x^2$ dari Polinomial Legendre

※ $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$

Pertimbangkan pula x^2 yang telah kita dapat dari $P_4(x)$

$$\Rightarrow -\frac{6}{7}x^2 + k\frac{3}{2}x^2 = 2x^2$$

$\frac{40}{21}P_2(x) = -\frac{20}{21}$

$$k\frac{3}{2}x^2 = \frac{20}{7}x^2$$

$$kx^2 = \frac{40}{21}x^2$$, maka $k = \frac{40}{21}$

-> Cari $-x$ dari Polinomial Legendre

※ $P_1(x) = x$

$$\Rightarrow -\frac{6}{5}x + nx = -x$$

$$nx = \frac{1}{5}x \longrightarrow \text{maka } n = \frac{1}{5}$$

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$\cdot\!>$ Cari 3 dari Polinomial Legendre

$P_0(x) = 1$

$$\frac{3}{35} - \frac{20}{21} + k = 3$$

$$k = 3 + \frac{20}{21} - \frac{3}{35}$$

$$= \frac{2205 + 700 - 63}{735}$$

$$= \frac{2842}{735} \times \frac{49}{49}$$

$$= \frac{58}{15}$$

Jadi, diperoleh bahwa :

$$x^4 + 2x^3 + 2x^2 - x + 3 = \frac{8}{35} P_4(x) + \frac{4}{5} P_3(x) + \frac{40}{21} P_2(x) + \frac{1}{5} P_1(x) + \frac{58}{15} P_0(x).$$

20). $f(x) = |x|, \quad -1 < x < 1$

Cari deret $P_n(x)$ apabila $f(x)$ dikontruksi !

▶ Gunakan formula koefisien deret Polinomial Legendre.

$\cdot\!>$ $a_0 = \frac{1}{2} \int_{-1}^{1} |x| \, dx$

$$= \frac{1}{2} \left[\int_{-1}^{0} -x \, dx + \int_{0}^{1} x \, dx \right]$$

$$= \frac{1}{2} \left[-\frac{1}{2} x^2 \Big|_{-1}^{0} + \frac{1}{2} x^2 \Big|_{0}^{1} \right]$$

$$= \frac{1}{2} \left[\frac{1}{2} + \frac{1}{2} \right]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

$\cdot\!>$ $a_1 = \frac{3}{2} \int_{-1}^{1} |x| . \, x \, dx$

$$= \frac{3}{2} \left[\int_{-1}^{0} -x^2 \, dx + \int_{0}^{1} x^2 \, dx \right]$$

$$= \frac{3}{2} \left[-\frac{1}{3} x^3 \Big|_{-1}^{0} + \frac{1}{3} x^3 \Big|_{0}^{1} \right]$$

$$= \frac{3}{2} \left[-\frac{1}{3} + \frac{1}{3} \right]$$

$$= 0$$

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$\cdot\!> a_2 = \frac{5}{2}\int_{-1}^{1} |x| . \frac{1}{2}(3x^2-1)\,dx$

$= \frac{5}{4}\left[\int_{-1}^{0} -x(3x^2-1)\,dx + \int_{0}^{1} x(3x^2-1)\,dx\right]$

$= \frac{5}{4}\left[\int_{-1}^{0} -3x^3+x\,dx + \int_{0}^{1} 3x^3-x\,dx\right]$

$= \frac{5}{4}\left[-\frac{3}{4}x^4+\frac{1}{2}x^2\Big|_{-1}^{0} + \frac{3}{4}x^4-\frac{1}{2}x^2\Big|_{0}^{1}\right]$

$= \frac{5}{4}\left[\frac{3}{4}-\frac{1}{2}+\frac{3}{4}-\frac{1}{2}\right]$

$= \frac{5}{4}\left[\frac{6}{4}-1\right]$

$= \frac{5}{4}\left[\frac{2}{4}\right]$

$= \frac{5}{8}$

$\cdot\!> a_3 = \frac{7}{2}\int_{-1}^{1} |x|\, \frac{1}{2}(5x^3-3x)\,dx$

$= \frac{7}{4}\left[\int_{-1}^{0} -x(5x^3-3x)\,dx + \int_{0}^{1} x(5x^3-3x)\,dx\right]$

$= \frac{7}{4}\left[\int_{-1}^{0} -5x^4+3x^2\,dx + \int_{0}^{1} 5x^4-3x^2\,dx\right]$

$= \frac{7}{4}\left[-x^5+x^3\Big|_{-1}^{0} + x^5-x^3\Big|_{0}^{1}\right]$

$= \frac{7}{4}\left[-(1-1)+(1-1)\right]$

$= 0$

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$$a_4 = \frac{9}{2} \int_{-1}^{1} |x| \cdot \frac{1}{8}(35x^4 - 30x^2 + 3)\, dx$$

$$= \frac{9}{16}\left[\int_{-1}^{0} -x(35x^4 - 30x^2 + 3)\, dx + \int_{0}^{1} x(35x^4 - 30x^2 + 3)\, dx\right]$$

$$= \frac{9}{16}\left[\int_{-1}^{0} -35x^5 + 30x^3 - 3x\, dx + \int_{0}^{1} 35x^5 - 30x^3 + 3x\, dx\right]$$

$$= \frac{9}{16}\left[-\frac{35}{6}x^6 + \frac{15}{2}x^4 - \frac{3}{2}x^2 \Big|_{-1}^{0} + \frac{35}{6}x^6 - \frac{15}{2}x^4 + \frac{3}{2}x^2 \Big|_{0}^{1}\right]$$

$$= \frac{9}{16}\left[-\left(-\frac{35}{6} + \frac{15}{2} - \frac{3}{2}\right) + \frac{35}{6} - \frac{15}{2} + \frac{3}{2}\right]$$

$$= \frac{9}{16}\left[\cancel{2} \cdot \frac{35}{\cancel{6}\,3} - \cancel{2} \cdot \frac{15}{\cancel{2}} + \cancel{2} \cdot \frac{3}{\cancel{2}}\right]$$

$$= \frac{9}{16}\left[\frac{35 - 45 + 9}{3}\right]$$

$$= \frac{9}{16}\left[-\frac{1}{3}\right]$$

$$= \frac{-3}{16}$$

Jadi, diperoleh :

$$f(x) = \frac{1}{2} P_0(x) + 0 . P_1(x) + \frac{5}{8} P_2(x) + 0\, P_3(x) + \left(-\frac{3}{16}\right) P_4(x) + \ldots$$

$$= \frac{1}{2} P_0(x) + \frac{5}{8} P_2(x) - \frac{3}{16} P_4(x) + \ldots$$

$$= \frac{8}{16} P_0(x) + \frac{10}{16} P_2(x) - \frac{3}{16} P_4(x) + \ldots$$

$$= \frac{1}{16}\left[8 P_0(x) + 10 P_2(x) - 3 P_4(x)\right] + \ldots$$

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